

Semi-shifted hybrid inflation with $B - L$ cosmic strings

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We discuss a new inflationary scenario which is realized within the extended supersymmetric Pati-Salam model which yields an acceptable b -quark mass for universal boundary conditions and $\mu > 0$ by modestly violating Yukawa unification and leads to new shifted, new smooth, or standard-smooth hybrid inflation. Inflation takes place along a “semi-shifted” classically flat direction on which the $U(1)_{B-L}$ gauge group remains unbroken. After the end of inflation, $U(1)_{B-L}$ breaks spontaneously and a network of local cosmic strings, which contribute a small amount to the curvature perturbation, is produced. We show that, in minimal supergravity, this “semi-shifted” inflationary scenario is compatible with a recent fit to data which uses field-theory simulations of a local string network. Taking into account the requirement of gauge unification, we find that, for spectral index $n_s = 1$, the predicted fractional contribution of strings to the temperature power spectrum at multipole $\ell = 10$ is $f_{10} \simeq 0.039$. Also, for $f_{10} = 0.10$, which is the best-fit value, we obtain $n_s \simeq 1.0254$. Spectral indices lower than about 0.98 are excluded and blue spectra are slightly favored. Magnetic monopoles are not formed at the end of semi-shifted hybrid inflation.

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I. INTRODUCTION

One of the most promising models for inflation [1] (for a review, see e.g. Ref. [2]) is, undoubtedly, hybrid inflation [3], which is [4, 5] naturally realized within supersymmetric (SUSY) grand unified theory (GUT) models. In the standard realization of SUSY hybrid inflation, the spontaneous breaking of the GUT gauge symmetry takes place at the end of inflation and, thus, superheavy magnetic monopoles [6] are copiously produced [7] if they are predicted by this symmetry breaking. In this case, a cosmological catastrophe is encountered.

In order to avoid this disaster, one can employ the smooth [7, 8] or shifted [9] variants of SUSY hybrid inflation (for a review, see Ref. [10]). In these inflationary scenarios, which, in their original realization, are based on non-renormalizable superpotential terms, the GUT gauge symmetry is broken to the standard model (SM) gauge group G_{SM} already during inflation and, thus, no magnetic monopoles are produced at the termination of inflation. New versions of these inflationary schemes can be implemented [11, 12] with only renormalizable superpotential terms within an extended SUSY GUT model based on the Pati-Salam (PS) gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$ [13], whose spontaneous breaking to G_{SM} predicts the existence of doubly charged [14] magnetic monopoles. Actually, this extended SUSY PS model was initially constructed [15] (see also Ref. [16]) for solving a very different problem. In SUSY models with exact Yukawa unification [17], such as the simplest SUSY PS model (see Ref. [18]), and universal boundary conditions, the predicted b -quark mass is [19] unaccept-

ably large for $\mu > 0$. However, it can be adequately reduced if Yukawa unification is moderately violated. This is achieved by extending the superfield content of the SUSY PS model so as to include, among other superfields, an extra pair of $SU(4)_c$ non-singlet $SU(2)_L$ doublets, which naturally mix [20] with the main electroweak doublets.

Fitting the recent data of the Wilkinson microwave anisotropy probe (WMAP) satellite with the standard power-law cosmological model with cold dark matter and a cosmological constant (Λ CDM), one obtains [21] values of the spectral index n_s which are clearly lower than unity. (Note, though, that some recent analyses, e.g. Ref. [22], reduce somewhat the evidence for $n_s < 1$.) However, in supergravity (SUGRA) with canonical Kähler potential, the above hybrid inflation models yield [11, 12, 23] n_s 's which are very close to unity or even larger than it although their running is negligible. This discrepancy may be resolved [24, 25, 26] by including non-minimal terms in the Kähler potential. Alternatively, if we wish to stick to minimal SUGRA, we can reduce [27] the spectral index predicted by the hybrid inflationary models by restricting the number of e-foldings suffered by our present horizon scale during the hybrid inflation which generates the observed curvature perturbations. The additional number of e-foldings required for solving the horizon and flatness problems of standard hot big bang cosmology can be provided by a subsequent second stage of inflation. In Ref. [28], we showed that the same extended SUSY PS model can lead to a two-stage inflationary scenario yielding acceptable n_s 's in minimal SUGRA. The first stage of inflation, during which the cosmological scales exit the horizon, is of the standard hybrid type, while the second stage, which provides the additional e-foldings, is of the new smooth hybrid type.

In this paper, we consider an alternative inflationary scenario which incorporates cosmic strings [29] (for a

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textbook presentation or a review, see e.g. Ref. [30]) and can also be naturally realized within this extended SUSY PS model with only renormalizable superpotential terms. As shown in Ref. [11], in a certain range of parameters, this model possesses a shifted classically flat direction along which $U(1)_{B-L}$ is unbroken. In order to distinguish it from the new shifted flat direction on which G_{PS} is broken to G_{SM} , we will call this flat direction “semi-shifted”. This direction acquires [31], as usual, a logarithmic slope from one-loop radiative corrections which are due to the SUSY breaking caused by the non-zero potential energy density on it. So, it can perfectly be used as an inflationary path along which “semi-shifted” hybrid inflation takes place. When the system crosses the critical point at which this path is destabilized, a waterfall regime occurs during which the $U(1)_{B-L}$ gauge symmetry breaks spontaneously and local cosmic strings are produced. The resulting string network can then contribute to the primordial curvature perturbations.

It has been argued [32], that, in the presence of a small contribution to the curvature perturbation from cosmic strings, the current cosmic microwave background (CMB) data can allow values of the spectral index that are larger than the ones obtained in the absence of strings. Therefore, we may hope that our semi-shifted hybrid inflationary scenario, which does involve cosmic strings, can be made compatible with the CMB data even without the use of non-minimal terms in the Kähler potential or a subsequent complementary stage of inflation. Recently, a fit to the CMB data and the luminous red galaxy data in the Sloan digital sky survey (SDSS) [33] on large length scales outside the non-linear regime was performed [34] by using field-theory simulations [35] of a dynamical network of local cosmic strings. It demonstrated that the Harrison-Zeldovich (HZ) model (i.e. $n_s = 1$) with a fractional contribution $f_{10} \approx 0.10$ from cosmic strings to the temperature power spectrum at multipole $\ell = 10$ is even moderately favored over the standard power-law model without strings. For the power-law Λ CDM cosmological model with cosmic strings, this fit yields [34] $n_s = 0.94 - 1.06$ and $f_{10} = 0.02 - 0.18$ at 95% confidence level (c.l.). We show that, under these circumstances, the semi-shifted hybrid inflationary model in minimal SUGRA can easily be compatible with the data. Note that there is obviously no formation of PS magnetic monopoles at the end of the semi-shifted hybrid inflation and, thus, the corresponding cosmological catastrophe is avoided.

In Sec. II, we summarize the salient features of the extended SUSY PS model and sketch the semi-shifted hybrid inflationary scenario with cosmic strings. In Sec. III, we calculate the one-loop radiative correction to the inflationary potential along the semi-shifted path. In Sec. IV, we include the minimal SUGRA correction to this inflationary potential, while Secs. V and VI refer to the inflation and string power spectrum respectively. Sec. VII is devoted to the presentation of our numerical results, which show that, in minimal SUGRA, our semi-shifted

hybrid inflationary scenario with cosmic strings can yield a spectral index close to unity and be compatible with the data. In Sec. VIII, we discuss briefly gauge unification. Finally, in Sec. IX, we present our conclusions.

II. SEMI-SHIFTED HYBRID INFLATION

We consider the extended SUSY PS model of Ref. [15], which can lead to a moderate violation of the asymptotic Yukawa unification [17] so that, for $\mu > 0$, an acceptable b -quark mass is obtained even with universal boundary conditions. The breaking of G_{PS} to G_{SM} is achieved by the superheavy vacuum expectation values (VEVs) of the right handed neutrino type components of a conjugate pair of Higgs superfields H^c and \bar{H}^c belonging to the $(\bar{4}, 1, 2)$ and $(4, 1, 2)$ representations of G_{PS} respectively. The model also contains a gauge singlet S and a conjugate pair of superfields $\phi, \bar{\phi}$ belonging to the $(15, 1, 3)$ representation of G_{PS} . The superfield ϕ acquires a VEV which breaks G_{PS} to $G_{SM} \times U(1)_{B-L}$. In addition to G_{PS} , the model possesses a Z_2 matter parity symmetry and two global $U(1)$ symmetries, namely a Peccei-Quinn and a R symmetry. Such continuous global symmetries can effectively arise [36] from the rich discrete symmetry groups encountered in many compactified string theories (see e.g. Ref. [37]). For details on the full field content and superpotential, the charge assignments, and the phenomenological and cosmological properties of this extended SUSY PS model, the reader is referred to Refs. [9, 15] (see also Ref. [16]). This model can lead to new shifted [12] and new smooth [11] hybrid inflation based solely on renormalizable interactions. It can also yield [28] a two-stage inflationary scenario consisting of hybrid inflation of the standard type followed by new smooth hybrid inflation.

The superpotential terms which are relevant for inflation are

$$W = \kappa S(M^2 - \phi^2) - \gamma S H^c \bar{H}^c + m \phi \bar{\phi} - \lambda \bar{\phi} H^c \bar{H}^c, \quad (1)$$

where M, m are superheavy masses of the order of the SUSY GUT scale $M_{GUT} \simeq 2.86 \times 10^{16}$ GeV and κ, γ, λ are dimensionless coupling constants. These parameters are normalized so that they correspond to the couplings between the SM singlet components of the superfields. In a general superpotential of the type in Eq. (1), M, m and any two of the three dimensionless parameters κ, γ, λ can always be made real and positive by appropriately redefining the phases of the superfields. The third dimensionless parameter, however, remains generally complex. For definiteness, we will choose here this parameter to be real and positive too as we did in Ref. [11].

The F-term scalar potential obtained from the superpotential W in Eq. (1) is given by

$$\begin{aligned} V = & |\kappa(M^2 - \phi^2) - \gamma H^c \bar{H}^c|^2 \\ & + |m\bar{\phi} - 2\kappa S\phi|^2 + |m\phi - \lambda H^c \bar{H}^c|^2 \\ & + |\gamma S + \lambda \bar{\phi}|^2 (|H^c|^2 + |\bar{H}^c|^2), \end{aligned} \quad (2)$$

where the complex scalar fields which belong to the SM singlet components of the superfields are denoted by the same symbol. We will ignore throughout the soft SUSY breaking terms [38] in the scalar potential since their effect on inflationary dynamics is negligible in our case as in the case of the conventional realization of shifted hybrid inflation.

From the potential in Eq. (2) and the vanishing of the D-terms (which implies that $\bar{H}^{c*} = e^{i\theta} H^c$), we find [11] that there exist two distinct continua of SUSY vacua:

$$\phi = \phi_+, \quad \bar{H}^{c*} = H^c, \quad |H^c| = \sqrt{\frac{m\phi_+}{\lambda}} \quad (\theta = 0); \quad (3)$$

$$\phi = \phi_-, \quad \bar{H}^{c*} = -H^c, \quad |H^c| = \sqrt{\frac{-m\phi_-}{\lambda}} \quad (\theta = \pi) \quad (4)$$

with $\bar{\phi} = S = 0$, where

$$\phi_{\pm} \equiv \frac{\gamma m}{2\kappa\lambda} \left(-1 \pm \sqrt{1 + \frac{4\kappa^2\lambda^2 M^2}{\gamma^2 m^2}} \right). \quad (5)$$

It has been shown [11] that the potential in Eq. (2), generally, possesses three flat directions. The first one is the usual trivial flat direction at $\phi = \bar{\phi} = H^c = \bar{H}^c = 0$ with $V = V_{\text{tr}} \equiv \kappa^2 M^4$. The second one, which appears at

$$\begin{aligned} \phi &= -\frac{\gamma m}{2\kappa\lambda}, \quad \bar{\phi} = -\frac{\gamma}{\lambda} S, \\ H^c \bar{H}^c &= \frac{\kappa\gamma(M^2 - \phi^2) + \lambda m\phi}{\gamma^2 + \lambda^2}, \\ V = V_{\text{nsh}} &\equiv \frac{\kappa^2\lambda^2}{\gamma^2 + \lambda^2} \left(M^2 + \frac{\gamma^2 m^2}{4\kappa^2\lambda^2} \right)^2, \end{aligned} \quad (6)$$

exists only for $\gamma \neq 0$ and is the trajectory for the new shifted hybrid inflation [12]. Along this direction, G_{PS} is broken to G_{SM} . The third flat direction, which exists only if $\tilde{M}^2 \equiv M^2 - m^2/2\kappa^2 > 0$, lies at

$$\phi = \pm \tilde{M}, \quad \bar{\phi} = \frac{2\kappa\phi}{m} S, \quad H^c = \bar{H}^c = 0. \quad (7)$$

It is a “semi-shifted” flat direction (in the sense that, although the field ϕ is shifted from zero, the fields H^c , \bar{H}^c remain zero on it) with

$$V = V_{\text{ssh}} \equiv \kappa^2(M^4 - \tilde{M}^4). \quad (8)$$

Along this direction G_{PS} is broken to $G_{\text{SM}} \times \text{U}(1)_{\text{B-L}}$.

In our subsequent discussion, we will concentrate on the case where $\tilde{M}^2 > 0$. It is interesting to note that, in this case, the trivial flat direction is [11] unstable as it is a path of saddle points of the potential, while the new smooth path, which exists for $\tilde{M}^2 < 0$ and was used in Ref. [11] as inflationary trajectory, disappears. Moreover, for $\tilde{M}^2 > 0$, we always have $V_{\text{ssh}} < V_{\text{nsh}}$. It is, thus, more likely that the system will eventually settle down on the semi-shifted rather than the new shifted flat direction. Semi-shifted hybrid inflation can then take place as the

system slowly rolls down the semi-shifted path driven by its logarithmic slope provided by one-loop radiative corrections [31] which are due to the SUSY breaking by the non-vanishing potential energy density on this path. As the system crosses the critical point of the semi-shifted path, the $\text{U}(1)_{\text{B-L}}$ gauge symmetry breaks generating a network of local cosmic strings, which contribute a small amount to the CMB temperature power spectrum. As mentioned, for models with local cosmic strings, it has been shown in Ref. [34] that, at 95% c.l., $n_s = 0.94 - 1.06$ and $f_{10} = 0.02 - 0.18$.

III. ONE-LOOP RADIATIVE CORRECTIONS

The one-loop radiative correction to the potential on the semi-shifted path is calculated by the Coleman-Weinberg formula [39]:

$$\Delta V = \frac{1}{64\pi^2} \sum_i (-1)^{F_i} M_i^4 \ln \frac{M_i^2}{\Lambda^2}, \quad (9)$$

where the sum extends over all helicity states i , F_i and M_i^2 are the fermion number and mass squared of the i th state and Λ is a renormalization mass scale. In order to use this formula for creating a logarithmic slope in the inflationary potential, one has first to derive the mass spectrum of the model on the semi-shifted path.

As mentioned, during semi-shifted hybrid inflation, the SM singlet components of ϕ , $\bar{\phi}$ acquire non-vanishing values and break G_{PS} to $G_{\text{SM}} \times \text{U}(1)_{\text{B-L}}$. The value of the complex scalar field S at a point of the semi-shifted path is taken real by an appropriate R transformation. For simplicity, we use the same symbol S for this real value of the field as for the complex field in general since the distinction will be obvious from the context. The deviation of the complex scalar field S from its (real) value at a point of the inflationary path is denoted by δS . We can further write $\phi = v + \delta\phi$, $\bar{\phi} = \bar{v} + \delta\bar{\phi}$ with $v = \pm\tilde{M}$, $\bar{v} = (2\kappa v/m)S$ and $\delta\phi$, $\delta\bar{\phi}$ being complex scalar fields. We can then define the canonically normalized complex scalar fields

$$\zeta = \frac{2\kappa v \delta S - m\delta\bar{\phi}}{(m^2 + 4\kappa^2 v^2)^{1/2}}, \quad \epsilon = \frac{m\delta S + 2\kappa v \delta\bar{\phi}}{(m^2 + 4\kappa^2 v^2)^{1/2}}. \quad (10)$$

We find that ϵ remains massless on the semi-shifted path. So, it corresponds to the complex scalar inflaton field $\Sigma = (mS + 2\kappa v\bar{\phi})/(m^2 + 4\kappa^2 v^2)^{1/2}$, which during inflation takes the form $\Sigma = (1 + 4\kappa^2 v^2/m^2)^{1/2} S$. Consequently, in our case, the real canonically normalized inflaton is

$$\sigma = 2^{1/2}(1 + 4\kappa^2 v^2/m^2)^{1/2} S, \quad (11)$$

where S is obviously rotated to be real.

Expanding the complex scalars ζ , $\delta\phi$, H^c , and \bar{H}^c in real and imaginary parts according to the prescription $\chi = (\chi_1 + i\chi_2)/\sqrt{2}$, we find that the mass-squared matrices M_-^2 of ζ_1 , $\delta\phi_1$, M_+^2 of ζ_2 , $\delta\phi_2$, M_1^2 of H_1^c , \bar{H}_1^c , and

M_2^2 of H_2^c , \bar{H}_2^c are given by

$$M_{\pm}^2 = m^2 \begin{pmatrix} 1 + a^2 & s(1 + a^2)^{1/2} \\ s(1 + a^2)^{1/2} & 1 + a^2 + s^2 \pm 1 \end{pmatrix}, \quad (12)$$

$$M_{1,2}^2 = m^2 \begin{pmatrix} s^2 b^2 & \mp b \\ \mp b & s^2 b^2 \end{pmatrix}, \quad (13)$$

where $a = 2\kappa v/m$, $b = (\gamma + \lambda a)/2\kappa$, and $s = 2\kappa S/m$. Note that the eigenvalues of the matrices M_{\pm}^2 are always positive. Though, this is not the case with $M_{1,2}^2$. Specifically, one of the two eigenvalues of each of these matrices is always positive, while the other one becomes negative for $|s| < s_c \equiv 1/\sqrt{|b|}$ (we assume that $b \neq 0$). This defines the critical point on the semi-shifted path at which this path is destabilized (see below).

The superpotential in Eq. (1) gives rise to mass terms between the fermionic partners of ζ , $\delta\phi$ and H^c , \bar{H}^c (the fermionic partner of ϵ remains massless). The squares of the corresponding mass matrices are found to be

$$M_0^2 = m^2 \begin{pmatrix} 1 + a^2 & s(1 + a^2)^{1/2} \\ s(1 + a^2)^{1/2} & 1 + a^2 + s^2 \end{pmatrix}, \quad (14)$$

$$\bar{M}_0^2 = m^2 \begin{pmatrix} s^2 b^2 & 0 \\ 0 & s^2 b^2 \end{pmatrix}. \quad (15)$$

This completes the analysis of the SM singlet sector of the model. In summary, we found four groups of two real scalars with mass-squared matrices M_+^2 , M_-^2 , M_1^2 , and M_2^2 and two groups of two Weyl fermions with mass matrices squared M_0^2 and \bar{M}_0^2 . The contribution of the SM singlet sector to the radiative corrections to the potential along the semi-shifted path is given by

$$\begin{aligned} \Delta V = & \frac{1}{64\pi^2} \text{Tr} \left\{ M_+^4 \ln \frac{M_+^2}{\Lambda^2} + M_-^4 \ln \frac{M_-^2}{\Lambda^2} \right. \\ & - 2M_0^4 \ln \frac{M_0^2}{\Lambda^2} + M_1^4 \ln \frac{M_1^2}{\Lambda^2} + M_2^4 \ln \frac{M_2^2}{\Lambda^2} \\ & \left. - 2\bar{M}_0^4 \ln \frac{\bar{M}_0^2}{\Lambda^2} \right\}. \end{aligned} \quad (16)$$

We now turn to the u^c , \bar{u}^c type fields which are color antitriplets with charge $-2/3$ and color triplets with charge $2/3$ respectively. Such fields exist in H^c , \bar{H}^c , ϕ , and $\bar{\phi}$ and we shall denote them by u_H^c , \bar{u}_H^c , u_ϕ^c , \bar{u}_ϕ^c , u_ϕ^c , and \bar{u}_ϕ^c . The relevant expansion of ϕ is

$$\begin{aligned} \phi = & \left[\frac{1}{\sqrt{12}} \begin{pmatrix} \mathbf{1}_3 & 0 \\ 0 & -3 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right] \phi \\ & + \begin{pmatrix} 0 & \mathbf{0}_3 \\ 1 & 0 \end{pmatrix} u_\phi^c + \begin{pmatrix} 0 & 1 \\ \mathbf{0}_3 & 0 \end{pmatrix} \bar{u}_\phi^c + \dots, \end{aligned} \quad (17)$$

where the SM singlet in ϕ (denoted by the same symbol) is also shown with the first (second) matrix in the brackets belonging to the algebra of $\text{SU}(4)_c$ ($\text{SU}(2)_R$). Here, $\mathbf{1}_3$ and $\mathbf{0}_3$ denote the 3×3 unit and zero matrices respectively. The fields u_ϕ^c , \bar{u}_ϕ^c are $\text{SU}(2)_R$ singlets, so only

their $\text{SU}(4)_c$ structure is shown and summation over their $\text{SU}(3)_c$ indices is implied in the ellipsis. The field $\bar{\phi}$ can be similarly expanded.

In the bosonic u^c , \bar{u}^c type sector, we find that the mass-squared matrices $M_{u\pm}^2$ of the complex scalar fields $u_{\chi\pm}^c = (u_\chi^c \pm \bar{u}_\chi^{c*})/\sqrt{2}$, for $\chi = H, \phi, \bar{\phi}$, are

$$M_{u+}^2 = m^2 \begin{pmatrix} c^2 s^2 - c & 0 & 0 \\ 0 & s^2 & -s \\ 0 & -s & 1 \end{pmatrix}, \quad (18)$$

$$M_{u-}^2 = m^2 \begin{pmatrix} c^2 s^2 + c & 0 & 0 \\ 0 & 2 + s^2 + \rho_g^2 & -s(1 - \rho_g^2) \\ 0 & -s(1 - \rho_g^2) & 1 + \rho_g^2 s^2 \end{pmatrix}, \quad (19)$$

where $c = (\gamma - \lambda a/3)/2\kappa$ and $\rho_g^2 = g^2 a^2/3\kappa^2$ with g being the G_{PS} gauge coupling constant. Note that ρ_g^2 parametrizes contributions arising from the D-terms of the scalar potential and M_{u+}^2 has one zero eigenvalue corresponding to the Goldstone boson which is absorbed by the superhiggs mechanism. Furthermore, one of the eigenvalues $m^2(c^2 s^2 \mp c)$ of the matrices in Eqs. (18) and (19) (depending on the sign of c) becomes negative as soon as s crosses below the point $s_c^{(1)} \equiv 1/\sqrt{|c|}$ on the semi-shifted path. So, if $s_c^{(1)}$ was larger than the critical value s_c , the system would be destabilized first in one of the directions $u_{H\pm}^c$. In this case, a $\text{SU}(3)_c$ -breaking VEV would develop. To avoid this, we should demand that $s_c^{(1)}$ is located lower than the critical point s_c , so that, after the end of inflation, the correct symmetry breaking is obtained. This gives the condition $|b| < |c|$, which we will consider later.

In the fermionic u^c , \bar{u}^c type sector, we obtain four Dirac fermions (per color): $\psi_{u_H^c}^D = \psi_{u_H^c} + \psi_{\bar{u}_H^c}^c$, $\psi_{u_\phi^c}^D = \psi_{u_\phi^c} + \psi_{\bar{u}_\phi^c}^c$, $\psi_{\bar{u}_H^c}^D = \psi_{\bar{u}_H^c} + \psi_{u_H^c}^c$, and $\psi_{\bar{u}_\phi^c}^D = \psi_{\bar{u}_\phi^c} + \psi_{u_\phi^c}^c$, and $-i\lambda^D = -i(\lambda^+ + \lambda^-)$. Here, ψ_χ is the fermionic partner of the complex scalar field χ and $\lambda^\pm = (\lambda^1 \pm i\lambda^2)/\sqrt{2}$, where λ^1 (λ^2) is the gaugino color triplet corresponding to the $\text{SU}(4)_c$ generators with $1/2$ ($-i/2$) in the $i4$ and $1/2$ ($i/2$) in the $4i$ entry ($i = 1, 2, 3$). The fermionic mass matrix is

$$M_{\psi_u} = m \begin{pmatrix} -cs & 0 & 0 & 0 \\ 0 & -s & 1 & -\rho_g \\ 0 & 1 & 0 & -\rho_g s \\ 0 & -\rho_g & -\rho_g s & 0 \end{pmatrix}. \quad (20)$$

To complete this sector, we must also include the gauge bosons $A^{1,2}$ which are associated with $\lambda^{1,2}$. They acquire a mass squared $M_g^2 = m^2 \rho_g^2 (1 + s^2)$.

The overall contribution of the u^c , \bar{u}^c type sector to ΔV in Eq. (9) is

$$\begin{aligned} \Delta V = & \frac{3}{32\pi^2} \text{Tr} \left\{ M_{u+}^4 \ln \frac{M_{u+}^2}{\Lambda^2} + M_{u-}^4 \ln \frac{M_{u-}^2}{\Lambda^2} \right. \\ & \left. - 2M_{\psi_u}^4 \ln \frac{M_{\psi_u}^2}{\Lambda^2} + 3M_g^4 \ln \frac{M_g^2}{\Lambda^2} \right\}. \end{aligned} \quad (21)$$

We will now discuss the contribution from the e^c, \bar{e}^c type sector consisting of color singlets with charge 1, -1 . Such fields exist in $H^c, \bar{H}^c, \phi, \bar{\phi}$ and we shall denote them by $e_H^c, \bar{e}_H^c, e_\phi^c, \bar{e}_\phi^c, e_{\bar{\phi}}^c, \bar{e}_{\bar{\phi}}^c$. The relevant expansion of ϕ is

$$\phi = \left[\frac{1}{\sqrt{12}} \begin{pmatrix} \mathbf{1}_3 & 0 \\ 0 & -3 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} e_\phi^c + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \bar{e}_\phi^c \right] \quad (22)$$

with the same notation as in Eq. (17). A similar expansion holds for $\bar{\phi}$. It turns out that the mass terms in this sector are exactly the same as in the u^c, \bar{u}^c type sector with $\lambda/3$ replaced by λ and $2g^2/3$ by g^2 . So, we will only summarize the results.

In the bosonic e^c, \bar{e}^c type sector, the mass-squared matrices $M_{e\pm}^2$ of the complex scalars $e_{\chi\pm}^c = (e_\chi^c \pm \bar{e}_\chi^{c*})/\sqrt{2}$, for $\chi = H, \phi, \bar{\phi}$, are

$$M_{e+}^2 = m^2 \begin{pmatrix} d^2 s^2 - d & 0 & 0 \\ 0 & s^2 & -s \\ 0 & -s & 1 \end{pmatrix}, \quad (23)$$

$$M_{e-}^2 = m^2 \begin{pmatrix} d^2 s^2 + d & 0 & 0 \\ 0 & 2 + s^2 + \tau_g^2 & -s(1 - \tau_g^2) \\ 0 & -s(1 - \tau_g^2) & 1 + \tau_g^2 s^2 \end{pmatrix}, \quad (24)$$

where $d = (\gamma - \lambda a)/2\kappa$ and $\tau_g = \sqrt{3/2}\rho_g$. Note that, again, M_{e+}^2 has one zero eigenvalue corresponding to the Goldstone boson which is absorbed by the superhiggs mechanism. Furthermore, one of the eigenvalues $m^2(d^2 s^2 \mp d)$ of the matrices in Eqs. (23) and (24) (depending on the sign of d) becomes negative as s crosses below $s_c^{(2)} \equiv 1/\sqrt{|d|}$ on the semi-shifted path. Therefore, we must impose the constraint $s_c^{(2)} < s_c \Rightarrow |b| < |d|$ for the same reason explained above.

In the fermionic e^c, \bar{e}^c type sector, we obtain four Dirac fermions with mass matrix

$$M_{\psi_e} = m \begin{pmatrix} -ds & 0 & 0 & 0 \\ 0 & -s & 1 & -\tau_g \\ 0 & 1 & 0 & -\tau_g s \\ 0 & -\tau_g & -\tau_g s & 0 \end{pmatrix}. \quad (25)$$

Finally, we again obtain two gauge bosons with mass squared $\hat{M}_g^2 = m^2 \tau_g^2 (1 + s^2)$.

The overall contribution of the e^c, \bar{e}^c type sector to ΔV in Eq. (9) is

$$\Delta V = \frac{1}{32\pi^2} \text{Tr} \left\{ M_{e+}^4 \ln \frac{M_{e+}^2}{\Lambda^2} + M_{e-}^4 \ln \frac{M_{e-}^2}{\Lambda^2} - 2M_{\psi_e}^4 \ln \frac{M_{\psi_e}^2}{\Lambda^2} + 3\hat{M}_g^4 \ln \frac{\hat{M}_g^2}{\Lambda^2} \right\}. \quad (26)$$

Let us now consider the d^c, \bar{d}^c type sector consisting of color antitriplets with charge $1/3$ and color triplets with charge $-1/3$. Such fields exist in $H^c, \bar{H}^c, \phi, \bar{\phi}$ and

we denote them by $d_H^c, \bar{d}_H^c, d_\phi^c, \bar{d}_\phi^c, d_{\bar{\phi}}^c, \bar{d}_{\bar{\phi}}^c$. The field ϕ can be expanded in terms of these fields as

$$\phi = \left[\begin{pmatrix} 0 & \mathbf{0}_3 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right] d_\phi^c + \left[\begin{pmatrix} 0 & 1 \\ \mathbf{0}_3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right] \bar{d}_\phi^c + \dots \quad (27)$$

with the notation of Eq. (17). The field $\bar{\phi}$ is similarly expanded.

In the bosonic d^c, \bar{d}^c type sector, the mass-squared matrices $M_{d\pm}^2$ of the complex scalars $d_{\chi\pm}^c = (d_\chi^c \pm \bar{d}_\chi^{c*})/\sqrt{2}$, for $\chi = H, \phi, \bar{\phi}$, are

$$M_{d\pm}^2 = m^2 \begin{pmatrix} e^2 s^2 \mp e & 0 & 0 \\ 0 & 1 + s^2 \mp 1 & -s \\ 0 & -s & 1 \end{pmatrix}, \quad (28)$$

where $e = (\gamma + \lambda a/3)/2\kappa$. Note that, again, one of the eigenvalues $m^2(e^2 s^2 \mp e)$ of these matrices (depending on the sign of e) becomes negative as s crosses below $s_c^{(3)} \equiv 1/\sqrt{|e|}$ on the semi-shifted path and we, thus, have to impose the constraint $s_c^{(3)} < s_c \Rightarrow |b| < |e|$, so that the correct symmetry breaking pattern occurs at the end of inflation.

In the fermionic d^c, \bar{d}^c type sector, we obtain three Dirac fermions (per color) with mass matrix

$$M_{\psi_d} = m \begin{pmatrix} -es & 0 & 0 \\ 0 & -s & 1 \\ 0 & 1 & 0 \end{pmatrix}. \quad (29)$$

Note that there are no D-terms, gauge bosons, or gauginos in this sector.

The contribution of this sector to ΔV in Eq. (9) is

$$\Delta V = \frac{3}{32\pi^2} \text{Tr} \left\{ M_{d+}^4 \ln \frac{M_{d+}^2}{\Lambda^2} + M_{d-}^4 \ln \frac{M_{d-}^2}{\Lambda^2} - 2M_{\psi_d}^4 \ln \frac{M_{\psi_d}^2}{\Lambda^2} \right\}. \quad (30)$$

Next, we consider the q^c, \bar{q}^c type fields which are color antitriplets with charge $-5/3$ and color triplets with charge $5/3$. They exist in $\phi, \bar{\phi}$ and we call them $q_\phi^c, \bar{q}_\phi^c, q_{\bar{\phi}}^c, \bar{q}_{\bar{\phi}}^c$. The relevant expansion of ϕ is

$$\phi = \left[\begin{pmatrix} 0 & \mathbf{0}_3 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \right] q_\phi^c + \left[\begin{pmatrix} 0 & 1 \\ \mathbf{0}_3 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \right] \bar{q}_\phi^c + \dots \quad (31)$$

and a similar expansion holds for $\bar{\phi}$.

In the bosonic q^c, \bar{q}^c type sector, the mass-squared matrices $M_{q\pm}^2$ of the complex scalars $q_{\chi\pm}^c = (q_\chi^c \pm \bar{q}_\chi^{c*})/\sqrt{2}$, for $\chi = \phi, \bar{\phi}$, are

$$M_{q\pm}^2 = m^2 \begin{pmatrix} 1 + s^2 \mp 1 & -s \\ -s & 1 \end{pmatrix}. \quad (32)$$

In the fermionic q^c, \bar{q}^c type sector, we obtain two Dirac fermions (per color) with mass matrix

$$M_{\psi_q} = m \begin{pmatrix} -s & 1 \\ 1 & 0 \end{pmatrix}. \quad (33)$$

There are no D-terms, gauge bosons, or gauginos in this sector as well.

The contribution of this sector to ΔV in Eq. (9) is

$$\Delta V = \frac{3}{32\pi^2} \text{Tr} \left\{ M_{q+}^4 \ln \frac{M_{q+}^2}{\Lambda^2} + M_{q-}^4 \ln \frac{M_{q-}^2}{\Lambda^2} - 2M_{\psi_q}^4 \ln \frac{M_{\psi_q}^2}{\Lambda^2} \right\}. \quad (34)$$

Finally, in $\phi, \bar{\phi}$ there exist color octet, $\text{SU}(2)_R$ triplet superfields: $\phi_8^0, \phi_8^\pm, \bar{\phi}_8^0, \bar{\phi}_8^\pm$, with charge 0, 1, -1 as indicated. The relevant expansion of ϕ is

$$\phi = \left[\begin{pmatrix} T_8 & 0 \\ 0 & 0 \end{pmatrix}, \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \phi_8^0 + \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \phi_8^+ + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \phi_8^- \right] + \dots, \quad (35)$$

where T_8 represents the eight $\text{SU}(3)_c$ generators appropriately normalized. A similar expansion holds for $\bar{\phi}$.

In the bosonic sector, we obtain two groups of 24 complex scalars, which can be combined in pairs of two with mass-squared matrix

$$M_{\phi_{8\pm}}^2 = m^2 \begin{pmatrix} 1 + s^2 \mp 1 & -s \\ -s & 1 \end{pmatrix}. \quad (36)$$

In the fermionic sector, we find 48 Weyl fermions which can be combined in pairs of two with mass matrix

$$M_{\psi_{\phi_8}} = m \begin{pmatrix} -s & 1 \\ 1 & 0 \end{pmatrix}. \quad (37)$$

The contribution of this sector to ΔV in Eq. (9) is

$$\Delta V = \frac{12}{32\pi^2} \text{Tr} \left\{ M_{\phi_{8+}}^4 \ln \frac{M_{\phi_{8+}}^2}{\Lambda^2} + M_{\phi_{8-}}^4 \ln \frac{M_{\phi_{8-}}^2}{\Lambda^2} - 2M_{\psi_{\phi_8}}^4 \ln \frac{M_{\psi_{\phi_8}}^2}{\Lambda^2} \right\}. \quad (38)$$

The final overall ΔV is found by adding the contributions from the SM singlet sector in Eq. (16), the u^c, \bar{u}^c type sector in Eq. (21), the e^c, \bar{e}^c type sector in Eq. (26), the d^c, \bar{d}^c type sector in Eq. (30), the q^c, \bar{q}^c type sector in Eq. (34), and the color octet sector in Eq. (38). These one-loop radiative corrections are added to the tree-level potential V_{ssh} yielding the effective potential along the semi-shifted inflationary path in global SUSY. They generate a slope on this path which is necessary for driving

the system toward the vacuum. The overall $\sum_i (-1)^{F_i} M_i^4$ is σ -independent, which implies that the overall slope of the effective potential is Λ -independent. This is a crucial property of the model since otherwise observable quantities like the power spectrum $P_{\mathcal{R}}$ of the primordial curvature perturbation or the spectral index would depend on the scale Λ , which remains undetermined.

Let us now discuss the constraints $0 < |b| < |c|, |d|, |e|$ derived in the course of the calculation of the mass spectrum on the semi-shifted path. It is easy to show that these constraints require that v be in one of the ranges

$$0 > v > -\frac{\gamma m}{2\kappa\lambda} \quad \text{or} \quad -\frac{\gamma m}{2\kappa\lambda} > v > -\frac{3\gamma m}{4\kappa\lambda}. \quad (39)$$

These two ranges of v lead, respectively, to the two different sets of SUSY vacua of Eqs. (3) and (4). To see this, let us replace all the fields in the scalar potential of Eq. (2) except H^c, \bar{H}^c by their values on the semi-shifted path. Taking into account that $\bar{H}^{c*} = e^{i\theta} H^c$ from the vanishing of the D-terms, we are then left with only two free degrees of freedom, namely $|H^c|$ and θ , and the potential becomes

$$V = V_{\text{ssh}} + 2m^2 b^2 \left(s^2 - \frac{\cos \theta}{b} \right) |H^c|^2 + (\gamma^2 + \lambda^2) |H^c|^4. \quad (40)$$

It is obvious from this equation that, if $b > 0$, which is the case in the first range for v in Eq. (39), the system will get destabilized toward the direction with $\cos \theta = 1$ leading to the SUSY vacua in Eq. (3), while, if $b < 0$, which holds in the second range for v in Eq. (39), the system will be led to the SUSY vacua in Eq. (4).

IV. SUPERGRAVITY CORRECTIONS

We now turn to the discussion of the SUGRA corrections to the inflationary potential of the model. The F-term scalar potential in SUGRA is given by

$$V = e^{K/m_{\text{P}}^2} \left[(F_i)^* K^{i*j} F_j - 3 \frac{|W|^2}{m_{\text{P}}^2} \right], \quad (41)$$

where K is the Kähler potential, m_{P} the reduced Planck mass, $F_i = W_i + K_i W/m_{\text{P}}^2$, a subscript i (i^*) denotes derivation with respect to the complex scalar field χ^i (χ^{i*}), and K^{i*j} is the inverse of the Kähler metric $K_{j i^*}$. We will consider SUGRA with minimal Kähler potential and show that the results of the fit in Ref. [34] can be naturally met.

The minimal Kähler potential in the model under consideration has the form

$$K^{\text{min}} = |S|^2 + |\phi|^2 + |\bar{\phi}|^2 + |H^c|^2 + |\bar{H}^c|^2 \quad (42)$$

and the corresponding F-term scalar potential is

$$V^{\text{min}} = e^{K^{\text{min}}/m_{\text{P}}^2} \left[\sum_{\chi} \left| W_{\chi} + \frac{W \chi^*}{m_{\text{P}}^2} \right|^2 - 3 \frac{|W|^2}{m_{\text{P}}^2} \right], \quad (43)$$

where χ stands for any of the five complex scalar fields appearing in Eq. (42). It is quite easily verified that, on the semi-shifted direction, this scalar potential expanded up to fourth order in $|S|$ takes the form (the SUGRA corrections to the location of the semi-shifted path are not taken into account since they are small)

$$V_{\text{ssh}}^{\text{min}} \simeq V_{\text{ssh}} e^{\frac{\tilde{M}^2}{m_{\text{P}}^2}} \left[1 + \frac{1}{2} \frac{\tilde{M}^2}{m_{\text{P}}^2} \frac{\sigma^2}{m_{\text{P}}^2} + \frac{1}{8} \left(1 + \frac{2\tilde{M}^2}{m_{\text{P}}^2} \right) \frac{\sigma^4}{m_{\text{P}}^4} \right], \quad (44)$$

where V_{ssh} is the constant classical energy density on the semi-shifted path in the global SUSY case and σ is the canonically normalized inflaton field defined in Eq. (11). Thus, after including the SUGRA corrections with minimal Kähler potential, the effective potential during semi-shifted hybrid inflation becomes

$$V_{\text{ssh}}^{\text{mSUGRA}} \simeq V_{\text{ssh}}^{\text{min}} + \Delta V \quad (45)$$

with ΔV representing the overall one-loop radiative correction calculated in Sec. III.

V. INFLATIONARY OBSERVABLES

The slow-roll parameters ε , η and the parameter ξ^2 , which enters the running of the spectral index, are given (see e.g. Ref. [40]) by

$$\varepsilon \equiv \frac{m_{\text{P}}^2}{2} \left(\frac{V'(\sigma)}{V(\sigma)} \right)^2, \quad (46)$$

$$\eta \equiv m_{\text{P}}^2 \left(\frac{V''(\sigma)}{V(\sigma)} \right), \quad (47)$$

$$\xi^2 \equiv m_{\text{P}}^4 \left(\frac{V'(\sigma)V'''(\sigma)}{V^2(\sigma)} \right), \quad (48)$$

where prime denotes derivation with respect to the real canonically normalized inflaton field σ defined in Eq. (11). Here and in the subsequent formulas in Eqs. (49) and (50), V is the effective potential $V_{\text{ssh}}^{\text{mSUGRA}}$ defined in Eq. (45). Inflation ends at $\sigma_f = \max\{\sigma_\eta, \sigma_c\}$, where $\sigma_\eta > 0$ denotes the value of the inflaton field when $\eta = -1$ and $\sigma_c > 0$ is the critical value of σ on the semi-shifted inflationary path corresponding to s_c .

The number of e-foldings from the time when the pivot scale $k_0 = 0.002 \text{ Mpc}^{-1}$ crosses outside the inflationary horizon until the end of inflation is (see e.g. Ref. [40])

$$N_Q \simeq \frac{1}{m_{\text{P}}^2} \int_{\sigma_f}^{\sigma_Q} \frac{V(\sigma)}{V'(\sigma)} d\sigma, \quad (49)$$

where σ_Q is the value of the inflaton field at horizon crossing of the scale k_0 . The inflation power spectrum $P_{\mathcal{R}}$ of the primordial curvature perturbation at the pivot scale k_0 is given (see e.g. Ref. [40]) by

$$P_{\mathcal{R}}^{1/2} \simeq \frac{1}{2\pi\sqrt{3}} \frac{V^{3/2}(\sigma_Q)}{m_{\text{P}}^3 V'(\sigma_Q)}. \quad (50)$$

The spectral index n_s , the tensor-to-scalar ratio r , and the running of the spectral index $dn_s/d\ln k$ are written (see e.g. Ref. [40]) as

$$n_s \simeq 1 + 2\eta - 6\varepsilon, \quad r \simeq 16\varepsilon, \\ \frac{dn_s}{d\ln k} \simeq 16\varepsilon\eta - 24\varepsilon^2 - 2\xi^2,$$

where ε , η , and ξ^2 are evaluated at $\sigma = \sigma_Q$. The number of e-foldings N_Q required for solving the horizon and flatness problems of standard hot big bang cosmology is approximately given (see e.g. Ref. [2]) by

$$N_Q \simeq 53.76 + \frac{2}{3} \ln \left(\frac{v_0}{10^{15} \text{ GeV}} \right) + \frac{1}{3} \ln \left(\frac{T_{\text{r}}}{10^9 \text{ GeV}} \right), \quad (51)$$

where $v_0 = V_{\text{ssh}}^{1/4}$ is the inflationary scale and T_{r} is the reheat temperature that is expected not to exceed about 10^9 GeV , which is the well-known gravitino bound [41]. In the following, we take T_{r} to saturate the gravitino bound, i.e. $T_{\text{r}} = 10^9 \text{ GeV}$.

VI. STRING POWER SPECTRUM

As mentioned before, the spontaneous breaking of the $U(1)_{\text{B-L}}$ gauge symmetry at the end of the semi-shifted hybrid inflation leads to the formation of local cosmic strings. These strings can contribute a small amount to the CMB power spectrum. Their contribution is parametrized [34] to a very good approximation by the dimensionless string tension $G\mu_s$, where G is the Newton's gravitational constant and μ_s is the string tension, i.e. the energy per unit length of the string. In Refs. [34, 35], local strings were considered within the Abelian Higgs model in the Bogomol'nyi limit, i.e. with equal scalar and vector particle masses. If this was the case in our model, the string tension would be given by

$$\mu_s = 4\pi |\langle H^c \rangle|^2, \quad (52)$$

where $\langle H^c \rangle$ is the VEV of H^c in the relevant SUSY vacuum and is responsible for the spontaneous breaking of the $U(1)_{\text{B-L}}$ gauge symmetry. However, as it turns out, the scalar-to-vector mass ratio in our model is somewhat smaller than unity. This is, though, not expected [42] to make any appreciable qualitative difference. Also, the strings in our model do not coincide with the strings in the simple Abelian Higgs model due to the presence of the field ϕ , which enters the string solution. We do not anticipate, however, that this will alter the picture in any essential way. Moreover, as one can show by using the results of Ref. [43], charged fermionic transverse zero energy modes do not exist in the presence of our strings, which, thus, do not exhibit fermionic superconductivity. Therefore, we will apply the results of Refs. [34, 35] in our model and adopt the formula in Eq. (52) for the string tension. This is certainly an approximation, but we believe that it is adequate for our purposes here. In

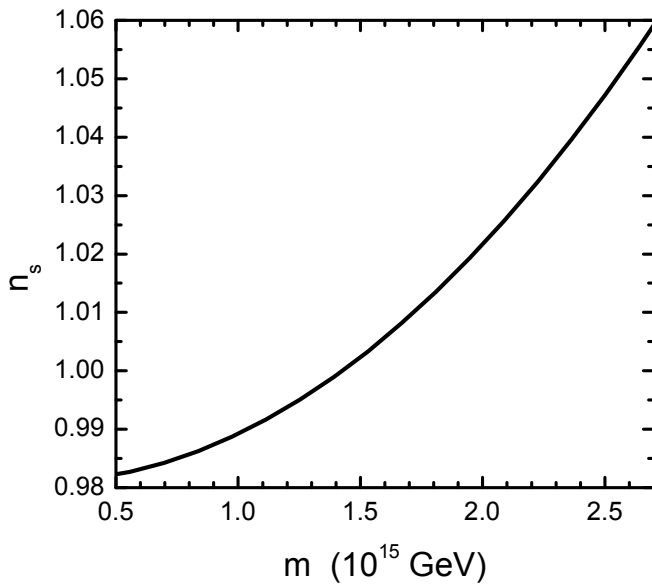


FIG. 1: Spectral index in semi-shifted hybrid inflation as a function of the mass parameter m in minimal SUGRA for $v = -\gamma m/4\kappa\lambda$, $\gamma/2\lambda = 1$, and $f_{10} = 0.10$.

Ref. [34], it was found that the best-fit value of the string tension required to normalize the WMAP temperature power spectrum at multipole $\ell = 10$ is

$$G\mu_s = 2.04 \times 10^{-6}. \quad (53)$$

This corresponds to $f_{10} = 1$, which is, of course, unrealistically large. The actual value of f_{10} is proportional to the actual value of $(G\mu_s)^2$. So, for any given value of f_{10} , we can calculate μ_s using its normalization in Eq. (53). From Eq. (52), we can then determine $|\langle H^c \rangle|$.

VII. NUMERICAL RESULTS

We choose the value v of the field ϕ on the semi-shifted path to lie in the first range for v in Eq. (39). In particular, we take it to be in the middle of this range, i.e.

$$v = -\frac{\gamma m}{4\kappa\lambda}. \quad (54)$$

This means, as we explained, that the universe will end up in the vacuum of Eq. (3). Similar results can be obtained if one chooses the value of v to be in the second range of Eq. (39). In order to fully determine the five parameters of the model, we need to make another four choices. One of them is taken to be the ratio $\gamma/2\lambda = 1$. Later we will comment on the dependence of the results on variations of this ratio, which is anyway weak. Secondly, we require the inflationary power spectrum amplitude of the primordial curvature perturbation at the pivot scale k_0 to have its central value [42] in the fit of Ref. [34]:

$$P_{\mathcal{R}}^{1/2} \simeq 4.47 \times 10^{-5}. \quad (55)$$

Further, we take, as an example, f_{10} to be equal to 0.10, its central value [34]. This determines $|\langle H^c \rangle|$ as discussed in Sec. VI. Finally, we calculate the spectral index for various values of the mass parameter m . The results are presented in Fig. 1, where m is restricted to be below 2.7×10^{15} GeV, so that the spectral index remains within its 95% c.l. range.

For m varying in the interval $(0.5 - 2.7) \times 10^{15}$ GeV, which is depicted in Fig. 1, the ranges of the various parameters of the model are: $M \simeq (0.6 - 3.5) \times 10^{15}$ GeV, $\gamma \simeq 0.029 - 0.914$, $\lambda \simeq 0.0145 - 0.457$, $\kappa \simeq 0.73 - 0.67$, $\sigma_Q \simeq (0.4 - 3.3) \times 10^{17}$ GeV, $\sigma_f \simeq (1.8 - 5.3) \times 10^{16}$ GeV, $N_Q \simeq 53.2 - 54.4$, $dn_s/d\ln k \simeq -(0.1 - 3.1) \times 10^{-6}$, $r \simeq (0.001 - 4.5) \times 10^{-5}$, and the ratio $\sigma_f/\sigma_c \simeq 2.6 - 7.7$. As one observes, we easily achieve spectral indices that are compatible with the fit of Ref. [34]. In particular, the best-fit value of the spectral index $n_s (= 1.00)$ is achieved for $m \simeq 1.40 \times 10^{15}$ GeV. However, indices lower than about 0.98 are not obtainable. Actually, as we lower m , the SUGRA corrections become less and less important and the spectral index decreases tending to its value (≈ 0.98) in global SUSY. In all cases, both the running of the spectral index and the tensor-to-scalar ratio are negligibly small.

Note that our results turn out to be quite sensitive to small changes of λ (and, thus, γ). This is due to the fact that the radiative correction to the inflationary potential contains logarithms with large positive as well as logarithms with large negative inclination with respect to σ . If no cancellation is assumed between these two competing trends, one ends up with either a rather fast rolling of the inflaton (dominance of logarithms with large positive inclination) or a negative inclination of the effective potential for large values of σ (dominance of logarithms with large negative inclination). In the latter case, after the inclusion of minimal SUGRA corrections, which lift the potential for $\sigma \gtrsim m_P$, a local minimum and maximum will be generated on the inflationary path. This leads [25, 26] to complications and must, therefore, be avoided. It turns out that a cancellation to the third significant digit between the positive and negative contributions to the derivative of the effective potential is needed in order to avoid these complications and ensure that the slow-roll conditions for the inflaton are fulfilled. This can be achieved by a mild tuning of the parameter λ to the third significant digit. So, the model entails a moderate tuning in one of its parameters in order to be cosmologically viable. Note, however, that this tuning needs only to be performed between the various contributions to the radiative correction and it is not spoiled by minimal SUGRA corrections. We should also mention that, in our model, σ_f turns out to be much larger than σ_c and inflation terminates well before the system reaches the critical point of the semi-shifted path. This is again due to the presence in the inflationary potential of logarithms with large inclination. Finally, we find that reducing the ratio $\gamma/2\lambda$ generally leads to a slight increase of the spectral index. Though, this dependence

is rather weak and that is why we have chosen to constrain this ratio to a constant value (instead of setting e.g. the ratio $\kappa/\lambda = \text{const.}$).

We observe numerically that, varying f_{10} within its 95% c.l. range $0.02 - 0.18$, the value of n_s changes only in the third decimal place. So, the curve in Fig. 1 is practically independent of f_{10} . We should, however, keep in mind that, for large values of m and low f_{10} 's, the constraint in Eq. (55) cannot be satisfied. Consequently, the curve in Fig. 1 applied to low values of f_{10} terminates on the right at a value of m which, of course, depends on f_{10} , but is, in any case, higher than about 2×10^{15} GeV.

We have seen that, in minimal SUGRA, the model develops a preference for values of m near 1.4×10^{15} GeV. On the other hand, for $f_{10} = 0.10$, the prediction for the value of m which is derived from gauge coupling constant unification is $m \simeq 2.085 \times 10^{15}$ GeV, as the reader may find out in Sec. VIII. However, one can see that, for this value of m , the predicted spectral index is $n_s \simeq 1.0254$, which lies inside the $1 - \sigma$ range for n_s given by the fit that we have been using in this paper.

VIII. GAUGE UNIFICATION

We will now discuss the question of gauge coupling constant unification in our model. As already mentioned, the VEVs of the fields H^c , \bar{H}^c break the PS gauge group G_{PS} to G_{SM} , whereas the VEV of the field ϕ breaks it only to $G_{\text{SM}} \times \text{U}(1)_{\text{B-L}}$. So, the gauge boson A^\perp corresponding to the linear combination of $\text{U}(1)_Y$ and $\text{U}(1)_{\text{B-L}}$ which is perpendicular to $\text{U}(1)_Y$ acquires its mass squared $m_{A^\perp}^2 = (5/2)g^2|\langle H^c \rangle|^2$ solely from the VEVs of H^c , \bar{H}^c . On the other hand, the masses squared m_A^2 and $m_{W_R}^2$ of the color triplet, antitriplet (A^\pm) and charged $\text{SU}(2)_R$ (W_R^\pm) gauge bosons get contributions from $\langle \phi \rangle$ too. Namely, $m_A^2 = g^2(|\langle H^c \rangle|^2 + (4/3)|\langle \phi \rangle|^2)$ and $m_{W_R}^2 = g^2(|\langle H^c \rangle|^2 + 2|\langle \phi \rangle|^2)$. Calculating the full mass spectrum of the model in the appropriate SUSY vacuum, one finds that there are fields acquiring mass of order m and others that acquire mass of order $g|\langle H^c \rangle|$. The presence of cosmic strings has forced the magnitude of the VEV of the fields H^c , \bar{H}^c in the SUSY vacuum to be in the range $(1.85 - 3.21) \times 10^{15}$ GeV (for $f_{10} = 0.02 - 0.18$), which is about an order of magnitude below the SUSY GUT scale. Furthermore, for all the values of the model parameters encountered here, the highest mass scale of the model in the SUSY vacuum is $m_{A^\perp} = \sqrt{5/2}g|\langle H^c \rangle|$. So, we set this scale equal to the unification scale M_x . From all the above, it is evident that the great desert hypothesis is not satisfied in this model and the simple SUSY unification of the gauge coupling constants is spoiled.

One can easily see that, although there exist many fields with $\text{SU}(3)_c$ and $\text{U}(1)_Y$ quantum numbers which can acquire heavy masses below the unification scale and, thus, affect the running of the corresponding gauge coupling constants, the only heavy fields with $\text{SU}(2)_L$ quan-

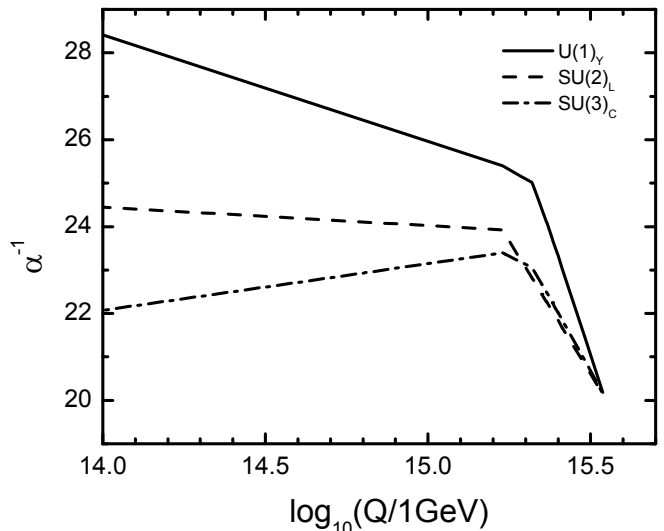


FIG. 2: Gauge coupling constant unification in our model for semi-shifted hybrid inflation in the case of minimal SUGRA for $v = -\gamma m/4\kappa\lambda$, $\gamma/2\lambda = 1$, and $f_{10} = 0.10$. The parameter α represents the three running SM fine structure constants as indicated and Q is the running mass scale.

tum numbers are h' and \bar{h}' belonging to the $(15,2,2)$ representation (see Ref. [15]). However, these fields affect equally the running of the $\text{U}(1)_Y$ gauge coupling constant and, consequently, cannot help us much in achieving gauge unification. We, therefore, assume that their masses are close to M_x so that they do not contribute to the renormalization group running. As a consequence of these facts, the $\text{SU}(2)_L$ gauge coupling constant fails to unify with the other gauge coupling constants. One is, thus, forced to consider the inclusion of some extra fields. There is a good choice using a single extra field, namely a superfield f belonging to the $(15,3,1)$ representation. This field affects mainly the running of the $\text{SU}(2)_L$ gauge coupling constant. If we require that f has charge $1/2$ under the global $\text{U}(1)_R$ symmetry, then the only renormalizable superpotential term in which this field is allowed to participate is a mass term of the form $\frac{1}{2}m_f f^2$. One can then tune the new mass parameter m_f , along with the mass m , so as to achieve unification of the gauge coupling constants. In contrast to Ref. [28], we will not include here the superpotential term $\phi^2 \bar{\phi}$ allowed [15] by the symmetries of the model since, as it turns out, it is not so useful in the present case. We will assume that the corresponding coupling constant is negligible.

We have implemented a code that is built on top of the **SOFTSUSY** code of Ref. [44] and performs the running of the gauge coupling constants at two loops. We have incorporated six mass thresholds below the unification scale M_x , namely m_f , m , $[m^2 + (4/3)\lambda^2|\langle H^c \rangle|^2]^{1/2}$, $[m^2 + 2\lambda^2|\langle H^c \rangle|^2]^{1/2}$, $g[|\langle H^c \rangle|^2 + (4/3)|\langle \phi \rangle|^2]^{1/2}$, and $g[|\langle H^c \rangle|^2 + 2|\langle \phi \rangle|^2]^{1/2}$. In Fig. 2, we present the unification of the SM gauge coupling constants in the $f_{10} = 0.10$ case. We deduce that gauge unification is achieved for

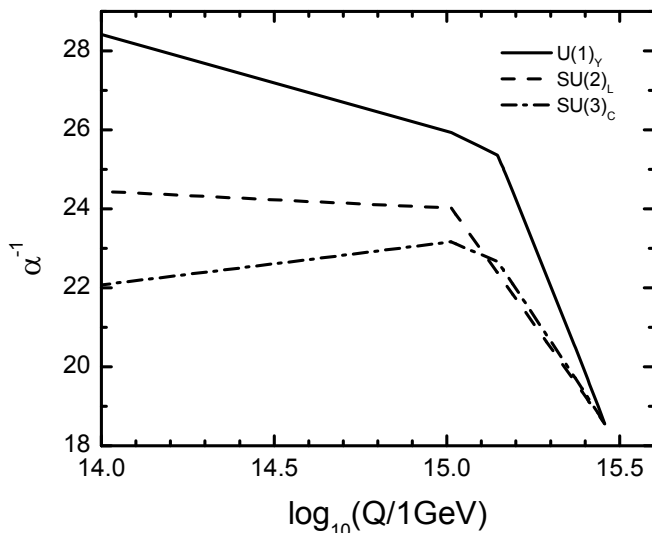


FIG. 3: Gauge coupling constant unification in our model for semi-shifted hybrid inflation in the case of minimal SUGRA for $v = -\gamma m/4\kappa\lambda$, $\gamma/2\lambda = 1$, and $n_s = 1$. Same notation as in Fig. 2.

$m_f \simeq 1.69 \times 10^{15}$ GeV and $m \simeq 2.085 \times 10^{15}$ GeV with the values of the other parameters of the model being $n_s \simeq 1.0254$, $M \simeq 2.53 \times 10^{15}$ GeV, $\gamma \simeq 0.515$, $\lambda \simeq 0.2575$, $\kappa \simeq 0.713$, $\sigma_Q \simeq 2.5 \times 10^{17}$ GeV, $\sigma_f \simeq 4.5 \times 10^{16}$ GeV, $N_Q \simeq 54.2$, $dn_s/d\ln k \simeq -0.8 \times 10^{-6}$, $r \simeq 1.5 \cdot 10^{-5}$, and the ratio $\sigma_f/\sigma_c \simeq 6.5$. The GUT gauge coupling constant turns out to be $g \simeq 0.789$ and the unification scale $M_x \simeq 3.45 \times 10^{15}$ GeV. In the HZ case (i.e. for $n_s = 1$), gauge unification is achieved for $m_f \simeq 1.025 \times 10^{15}$ GeV and $m \simeq 1.40 \times 10^{15}$ GeV (see Fig. 3), which corresponds to $f_{10} \simeq 0.039$, $M \simeq 1.68 \times 10^{15}$ GeV, $\gamma \simeq 0.367$, $\lambda \simeq 0.1835$, $\kappa \simeq 0.721$, $\sigma_Q \simeq 1.5 \times 10^{17}$ GeV, $\sigma_f \simeq 3.4 \times 10^{16}$ GeV, $N_Q \simeq 53.9$, $dn_s/d\ln k \simeq -0.2 \times 10^{-6}$, $r \simeq 0.3 \times 10^{-5}$, $\sigma_f/\sigma_c \simeq 6.3$, $g \simeq 0.823$, and $M_x \simeq 2.865 \times 10^{15}$ GeV. Note that the unification scale M_x turns out to be somewhat small. This fact, however, does not lead to unacceptably fast proton decay since the relevant diagrams are suppressed by large factors (for details, see Ref. [9]).

IX. CONCLUSIONS

It has been shown that the extension of the SUSY PS model which has been introduced in Ref. [15] in order to solve the b -quark mass problem in SUSY GUTs with exact asymptotic Yukawa unification, such as the simplest SUSY PS model, universal boundary conditions and $\mu > 0$ is a very fruitful framework for constructing hybrid inflationary models. Indeed, it has been demonstrated that this model automatically and naturally leads to the so-called new shifted and new smooth hybrid inflationary scenarios, which are based only on renormalizable superpotential terms and avoid the cosmological

disaster from a possible overproduction of PS magnetic monopoles at the termination of inflation. These variants of SUSY hybrid inflation, however, yield, in the context of minimal SUGRA, values of the spectral index which lie above the range allowed by the recent CMB data. It is quite remarkable that this problem can also be resolved within the same extended SUSY PS model by considering a two-stage inflationary scenario without the need of non-minimal terms in the Kähler potential.

Here, we have presented an alternative inflationary scenario which can also be naturally realized within the same PS model using the same renormalizable superpotential terms as in the previous inflationary scenarios and is compatible with the recent data within minimal SUGRA. This scenario incorporates cosmic strings produced at the end of inflation. Our PS model, in a certain range of parameters, possesses a semi-shifted classically flat direction on which the PS gauge group is spontaneously broken only to $G_{\text{SM}} \times U(1)_{\text{B-L}}$. This direction acquires a slope from one-loop radiative corrections originating from the SUSY breaking caused by the non-zero potential energy density on this trajectory. Therefore, it can be used as an inflationary path. We coined the name semi-shifted hybrid inflation for the resulting inflationary scenario. As it turns out, inflation terminates by violating the slow-roll conditions well before the system reaches the critical point of the semi-shifted path. Subsequently, the system crosses the critical point where the semi-shifted trajectory is destabilized and the spontaneous breaking of the $U(1)_{\text{B-L}}$ gauge symmetry takes place. As a result of this breaking, a network of local cosmic strings, which can contribute to the primordial curvature perturbation, is produced.

It is known that, in the presence of a network of cosmic strings, the present CMB data can easily become compatible with values of the spectral index which are close to unity or even exceed it. We use a recent fit [34] to CMB and SDSS data which is based on field-theory simulations of a dynamical network of local cosmic strings. For the power-law Λ CDM cosmological model, this fit implies that, at 95% c.l., the spectral index is $n_s = 0.94 - 1.06$ and the fractional contribution of cosmic strings to the temperature power spectrum at $\ell = 10$ is $f_{10} = 0.02 - 0.18$. Our numerical results show that the semi-shifted hybrid inflationary model can easily become compatible with this fit without the need of non-minimal terms in the Kähler potential or a subsequent second stage of inflation. Taking into account the constraints from the unification of the gauge coupling constants, we have found that, for a certain choice of parameters, the model yields $f_{10} \simeq 0.039$ in the HZ case (i.e. for $n_s = 1$) and $n_s \simeq 1.0254$ for the best-fit value of f_{10} ($= 0.10$). Spectral indices which are lower than about 0.98 cannot be obtained. So, the model shows a slight preference to blue spectra. The cosmological disaster from the possible overproduction of PS magnetic monopoles is avoided since there is no production of such monopoles at the end of the semi-shifted hybrid inflation.

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